INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 40, Northern Spring 2019 (A Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. The King gives the following task to his two wizards. The first wizard should choose 7 distinct positive integers whose sum is 100 and secretly let the King know the numbers. Also, the first wizard should tell the second wizard the fourth largest number only. The second wizard must then tell the King all the chosen numbers correctly. Can the wizards succeed for sure? The wizards cannot discuss their strategy beforehand. (5 points)

2. There are 2019 crickets sitting on a straight line. Consider each cricket to be a point on the line. At each move one of the crickets jumps over one of the other crickets, landing at a point that is the same distance away from that cricket as before the jump. Jumping to the right only, the crickets are able to position themselves so that some pair of them are located exactly 1 mm from each other. Prove that the crickets are also able to position themselves so that two of them are exactly 1 mm apart, with the crickets jumping to the left only, and starting from the same initial position. (7 points)

3. Two equal non-intersecting wooden discs, one coloured grey and the other black, are in a fixed position of the plane. A wooden triangle with one grey edge and one black edge can be moved in the plane so that the discs remain outside the triangle while the coloured edges of the triangle are tangent to the discs of the same colour (the points of tangency not being the vertices). Prove that the line that contains the angle bisector of the angle between the grey and black edges of the triangle always passes through a fixed point of the plane, no matter where the triangle is located. (7 points)

4. A regular 100-gon is given, and all possible diagonals are drawn. Each line segment now drawn, is either coloured red, if it is an edge of the 100-gon or if the number of vertices of the 100-gon between its endpoints is even, or is coloured blue, otherwise. A number is assigned to each vertex so that the sum of the squares of the numbers on all 100 vertices is equal to 1. The product of the numbers at the endpoints of each line segment is assigned to that line segment. Then, the sum of the numbers assigned to the blue line segments is subtracted from the sum of the numbers assigned to the red line segments. What is the largest number that could result from performing this subtraction? (8 points)

Continued over the page

5. It is desired that all the squares of an $n \times n$ table ($n > 1$) be filled with distinct integers from 1 to n^2, such that there is one number per square, and each pair of consecutive integers are placed in squares that share a side, while any pair of integers having the same remainder on division by n are placed in distinct rows and distinct columns. For which n is this possible? (9 points)

6. Let ABC be an isosceles triangle with point K inside so that $CK = AB = BC$ and $\angle KAC = 30°$. Determine the size of $\angle AKB$. (9 points)

7. There are 100 piles of stones, with 400 stones in each pile. On each move Petya must choose two piles and remove one stone from each of them. Then, he is awarded a number of points equal to the non-negative difference of the numbers of stones in the two modified piles. The task is complete when Petya has removed all stones. What is the largest total number of points Petya can be awarded, if his initial score is 0? (12 points)